Name: ___________________________ Date: ________

QUIZ

Calculus: Integration 2

Applications of the Definite Integral:

Calculate Area Problems

<u>Directions:</u>

You have 20 minutes to find the solution/Area. Use the "Integral Calculus Concept", its formal definition, rules of finding Integrals, methods to integrate, etc.

Draw the areas bounded by curves, volumes by slicing, volumes of solids of revolution, and the lenghts of arcs of a curve.

When Trigonometry, integrate, find integrals, and/or sketch the graph of the function f(x). Analyze the Graph when appropriate.

Pay close attention to the given Hints.

Grade: ______

Teacher's Signature: ________________________

1. Areas bounded by curves.

The area of a region bounded by the graph of a function, the x-axis, and the two vertical boundaries can be determined directly by evaluating a definite integral.

Find the area (A) of the region lying below the graph of f(x), above the x-axis, between the lines x = a and x = b.

Hint:
Draw the graph y = f(x).
Determine the boundaries (i.e., close interval [a, b]).
a. If f(x) is greater than and equal to zero on [a, b].
b. If f(x) is less than and equal to zero on [a, b].
c. If y = x square, the x-axis, x = -2, and x = 3

2. Find the area (A) of the region bounded by

$y = x^3 + x^2 - 6x$ and the x-axis

<u>Hint:</u>
Determine where the graph intersects the x-axis by making y = 0.
Draw the curve. If x cube then the x-axis is crossed at three different points.
f(x) greater than and equal to 0 on [a, c] and f(x) less than and equal to 0 on [c, b].
Calculate the Area (A) by solving the Integral

3. Find the area (A) bounded by

$y = x^2$ and $y := 8 - x^2$

<u>Hint:</u>
Determine where the two graphs intercept, by solving the two equations y = y.
f(x) greater than and equal to 0 and g(x) less than and equal to 0 on [a, b].
Calculate the area (A) by solving the Integral.

4. Areas and Volumes of solids with known cross sections.

The definite integral can be used to find the volume of a solid with specific cross sections on an interval, provided we know a formula for the region determined by each cross section.

If the cross sections generated are penperdicular to the x-axis, then the areas will be functions of x, denoted by A(x). The volume (V) of the solid on the interval [a, b] is

$$V = \int_a^b A(x)\,dx$$

Find the volume of the solid whose base is the region inside the circle

$x^2 + y^2 = 9$ If cross sections taken perpendicular to the y-axis are squares.

Hint:
draw the graph (i.e., a circle).
Draw the square cross section.
Area (A) of an arbitrary square cross section is A = S square, where

$$s = 2\sqrt{9 - y^2}$$

Calculate the area (A)
Calculate the volume (V) by using the Integral on [-3, 3] (radius of the circle)