Name: ________________________ Date: _______

Test

Applications of the Derivative

Problems - "Rate of Change" applied to Biology, Physics, Engineering, Chemistry

Find the Solution

<u>Directions:</u>

You have 45 minutes to apply the concept of the Derivative.

Use initial values; differentiate the equations. Derive the equation. Clearly indicate the "steps" to solve the problems. Draw the picture and analyze it. Use the Chain Rule (Apply Leibnitz Notation concept). For example, IF V = f(r(t)) THEN dV/dt = dV/dr . dr/dt.

Grade: ______

Teacher's Signature: __________________________

1. Biology:

Problem: Suppose that a bacteria population starts with 500 bacteria and triples every hour.

a.) What is the bacteria growth after 3 hours? 4 hours? t hours?
b.) Using the derivative of the exponential function (growth), estimate the rate of increase of the bacteria growth after 6 hours.

Solution: Hint
1. visualize the problem.
2. given: initial conditions = 500
3. given: grow triples every hour - the bacteria growth is increasing by 3 (triple) - find the equation:

The rate of growth of the bacteria population at time t is:

$$\frac{d}{dt}n = \frac{d}{dt}\left(No\,3^{t}\right) = No(1.1)\,3^{t}$$

where No = initial condition
No = 500 bacteria

2. Biology: "rate of change" of a healing wound

Problem: The Area of a healing wound is given by

$$A := \pi r^2$$

The radius is decreasing at the rate of 2 milimeters per day (-2 mm/day) at the moment when r = 25 mm. How fast is the Area decreasing at that moment.

a.) Calculate the initial conditions (at that instant).
b.) How long does the wound take to heal.

Solution: Hint
1. visualize the problem.
2. calculate the initial conditions.
3. Chain Rule (Leibnitz Notation) A = f(r(t)) dA/dt = dA/dr . dr/dt.
4. how long does the wound take to heal, if it is decreasing by 2 (double) - find the equation:

$$\frac{d}{dt}n = \frac{d}{dt}\left(No\,2^t\right) = No(0.69)\,2^t$$ where No = initial condition

3. Physics Or, Architecture: Finding Maxima and Minima

1. draw the picture and analyze it.
2. find critical points (d/dx = 0).
3. find the Maxima (d2/dx2 - second derivative)

Problem: A cone with slant height of 6 inches is to be constructed. What is the largest possible volume of such a cone?

Solution:

1. Draw the cone and place the variables in it.
2. Given V = 1/3 PI r2 h volume of the cone.
3. Get V = f(h) Apply Pythagoras calculate r = f(h)
4. Find critical points V'(h) = 0 Then Maxima V"(h)

4. Physics: Quantities related by more than one equation

Problem: An ice cube is melting. When the volume of the cube is 8 cm3, it is melting at a rate of 4 cm3/sec. Find the rate of change of the surface area of the cube at that moment.

Solution:
1. Draw the cube and place the variables in it.
2. Given V = base area x hight "rate of change" of the cube.
3. Question: to find the rate of change of the "Surface".
   Not V (Volume), but S (Surface) of the cube.
4. Use Chain Rule (Leibnitz Notation) $V = f(x(t))$ $dV/dt = dV/dx \cdot dx/dt$.
5. Find ds/dt

5. Physics:

Problem: A particle moves according to the law of motion

$$s = f(t) = t^3 - 12t^2 + 36t \qquad t \geq 0$$

where t is measured in seconds and s in meters.

a.) Find the velocity at time t.
b.) What is the velocity after 3 seconds?
c.) When is the particle at rest?
d.) When is the particle moving forward?
e.) Find the total distance traveled during the first 8 seconds.
f.) find the acceleration at time t and after 3 seconds.

Solution:
1. visualize the problem (draw and place the variables in it).
2. Given S (space) - the particle moving in a straight line.
3. Question: calculate velocity and aceleration.
4. when the particle is at rest... v = 0.

6. Physics:

Problem: If a ball is thrown vertical upward with a velocity of 80 ft/sec, then its height after t seconds is

$$s = f(t) = 80\,t - 16\,t^2$$

a.) What is the maximum height reached by the ball?
b.) What is the velocity of the ball when it is 96 feet above the ground on its way up?

Solution:
1. visualize the problem (draw and place the variables in it).
2. Given S (space) - the particle moving in a straight line.
3. Question: calculate velocity and aceleration.
4. when the particle is at rest... v = 0.

7. Biology:

Problem: The volume of a growing spherical cell is

$$v = \frac{4}{3} \pi r^3$$

where the radius r is measured in micrometers.

a.) Find the average rate of change of V with respect to R when R changes from 5 to 8 micrometers.
b.) Find the instantaneous rate of change of V with respect to R when R is 5 micrometers.

Solution:
1. visualize the problem (draw and place the variables in it).
2. Given V (volume) of the blodd cell.
3. Question: calculate V = f(r) derivative V'.

8. Chemistry:

Problem: If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torriceli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \qquad 0 \le t \le 40$$

Find the rate at which water is draining from the tank after
a) 5 minutes, b) 10 minutes, c) 20 minutes and d) 40 minutes
At what time is the water flowing out the fastest? The slowest?

Solution:
1. visualize the problem (draw and place the variables in it).
2. Given V (volume) of the tank.
3. Question: calculate V = f(t) derivative V'.

9. <u>Chemistry:</u>

<u>Problem:</u> Boyle's Law states that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant: PV = C.
a.) Find the rate of change of volume with rspect to pressure.
b.) A sample of gas is in a container at low pressure and is steadily compressed at constant temperature for 10 minutes. Is the volume decreasing more rapidly at the beginning or the end of the 10 minutes?
c.) Prove that the isotherm compressibility is given by B = 1/P.

HINT: The volume V (in cubic meters) of a sample of air at 25 centigrades was found to be related to the pressure P (in kilopascals) by the equation:

$$V = \frac{5.3}{P}$$

As P increases, V decreases, so dV/dP < 0.

The compresibility is defined by introducing a minus sign and dividing this derivative by the volume V:

$$\text{isothermal compressibility} \quad = \quad \beta = \frac{-1}{V} \cdot \frac{d}{dP} V$$

<u>Solution:</u>
1. visualize the problem (draw and place the variables in it).
2. Given V (volume) of the blodd cell.
3. Question: calculate V = f(r) derivative V'.